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Generalizations of the Faraday problem in mechanical system "reservoir–liquid with a free surface"

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Two generalizations of the classical Faraday problem on development of parametric resonance in mechanical system "reservoir – liquid with free surface", namely, the effect of supplementary degree of freedom, i.e., possibility of horizontal motion of reservoir due to transversal motion of free surface of liquid, and effect of supplementary degree of freedom, i.e., possibility of angular oscillations of reservoir, which is suspended as pendulum, due to transversal oscillations of a free surface of liquid. Investigation is done on the basis of efficient nonlinear multimodal model, which considers combined motion of reservoir and free surface of the liquid. It was shown that, in contrast to the classical Faraday problem, dynamical processes in the system are developed as aggregate of parametric and forced mechanisms of oscillations. For the considered generalizations of the Faraday problem transition of oscillations of free surface of the liquid into nonlinear range of excitations is possible for any frequency of external vertical excitation of reservoir

Keywords: *nonlinear dynamics of liquid, free surface, parametric resonance, Faraday problem, transition to stationary mode*

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1. Introduction

Parametric resonance in the mechanical system "reservoir – free surfaced liquid" was investigated for the first time by Faraday in 1831. Cylindrical reservoir, partially filled by water, was installed on special laboratory equipment and was capable to perform motion in vertical direction according to the prescribed law. As the result of the experiment Faraday ascertained that first resonance frequency of the free surface of liquid is equal to a half of the frequency of perturbation of the reservoir. This result is known in history of mechanics as the Faraday classical problem about parametric oscillations of free surface of liquid in reservoir on movable foundation. (Fig. 1).

Starting from discovering this effect great number of investigations were done dealing with theoretical and applied aspects of the phenomenon. The most complete survey of these publications published before 2005 was stated in monograph [5]. Among recent publications in this area it is necessary to note art5icles by T.Ikeda, in particular [6]. Both theoretical results on investigations of conditions of origination of parametric resonance on the basis of the Van der Pole method and experimental ones, connected with process of development of oscillations with amplitude and phase modulation and transition of the system on mode of chaotic oscillations were adduced.

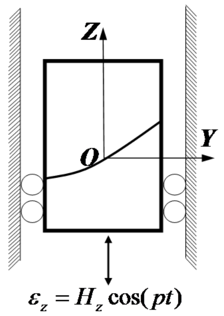

Fig. 1. Mechanical schematic diagram of the classical Faraday problem.

Since in the classical Faraday problem, reservoir moves only vertically according to the given law, liquid oscillations have no effect on on the character of its motion. Actually this means that reservoir moves in vertical channel or has infinitely great mass. However, in most practical applications (rolling and pitching of ships on waves, flight of rockets and launchers, etc.) structure with liquid can perform translational and rotational motions in different planes because of oscillations of liquid with a free surface and due to external force and moment loading. Here the liquid mass can considerably exceed the mass of reservoir, therefore, taking into account of combined motion of reservoir and free surfaced liquid and their interaction is the crucial factor in these problems.

Supplementary effects account in the classical Faraday problem about development of parametric resonance on a free surface of liquid makes it possible to create the following classification of generalizatons of the Faraday problem:

— reservoir moves vertically according to the prescribed harmonic law in the field of weak gravity; under these conditions it is necessary to take into account surface tension forces on a free surface of liquid;
— reservoir moves vertically according to the prescribed harmonic law and can perform translational motions in horizontal plane due to antisymmetric oscillations of liquid free surface (introduction of supplementary degree of freedom into the system, namely, potential of reservoir motion in the horizontal plane) (Fig. 2);
— reservoir is fixed on pendulum suspension, the point of suspension moves vertically точка according to the given harmonic law, reservoir can perform angular motion due to antisymmetric oscillations of a free surface of liquid (introduction of supplementary degree of freedom into the system, namely, potential of angular motion of reservoir) (Fig. 3);
— reservoir moves vertically, however not according to the given harmonic law, but under action of harmonic force applied to reservoir (combined statement of the problem);
— reservoir moves vertically under action of harmonic force and can perform translational motion in horizontal plane due to antisymmetric oscillations of a free surface of liquid;
— reservoir moves vertically under action of harmonic force and can perform angular motion in the horizontal plane due to antisymmetric oscillations of a free surface of liquid;

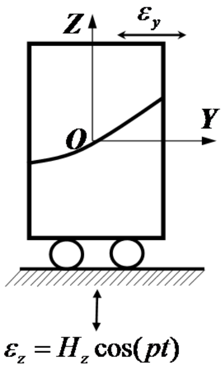

Fig. 2. Mechanical schematic diagram of the generalized Faraday problem with potential of horizontal motion of reservoir.

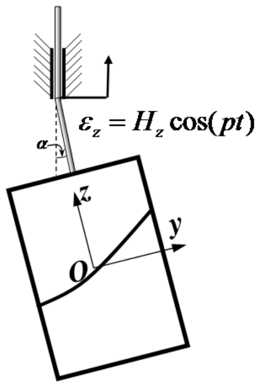

Fig. 3. Mechanical schematic diagram of the generalized Faraday problem with potential of angular motion of reservoir.

For analysis of some common used hypotheses in this problem and determination of validity range of the developed approach we analyzed all mentioned problems under the presence of liquid viscosity.

In the present article we investigate theoretically generalization of the Faraday problem for two variants of the stated above classification, namely, 1) reservoir moves vertically according to the given harmonic law and can perform translation motions in horizontal plane due to antisymmetric oscillations of a free surface of liquid (Fig. 2); 2) reservoir is fixed on pendulum suspension, the point of suspension moves vertically according to harmonic law due to antisymmetric oscillations of a free surface of liquid (Fig. 3). Moreover, for providing completeness of problem statement we refuse usage of the hypothesis on potential of elimination of neglecting oscillations on normal frequencies of the system, which is used by majority of researchers in this field (modern experimental studies showed that account of oscillations of liquid free surface on normal and combination frequencies is determinative [4,7]); investigation of dynamics of system on the basis of nonlinear mathematical multimode model (12 normal modes of oscillations); problem statement for couple motion of reservoir and liquid [2].

2. Mathematical model of the mechanical system

Let us consider a cylindrical reservoir with absolutely rigid walls, partially filled with liquid. We suppose liquid to be ideal, incompressible, homogeneous and its initial motion is vortex free. Investigation of peculiarities of parametric resonance in the generalized Faraday problem will be done on the basis of the mathematical model, developed in [2].

Let us introduce conventionally immovable reference frame $O_1X_1X_2X_3$, reference frame $Oxyz$ fixed with reservoir and reference frame $OY_1Y_2Y_3$ with origin at the point O, whose axes are correspondingly parallel to axes of the reference frame $O_1X_1X_2X_3$ (Fig. 4). Point O is at center of unperturbed free surface of liquid, axis Oz is directed toward external normal to unperturbed free surface of liquid. Motion of the point O in reference frame $O_1X_1X_2X_3$ is determined by the radius-vector $\boldsymbol{\varepsilon}(t)$, while rotational motion of reference frame $Oxyz$ relative to $O_1X_1X_2X_3$ is determined by three angles of turn $\alpha_1, \alpha_2, \alpha_3$. Here angle α_1 is defined as angle of rotation of the reference frame $Oxyz$ about the axis OY_1, angle α_2 angle of rotation of the system relative to new position of OY_2, and α_3 is angle of rotation of the system relative to new position of OY_3.

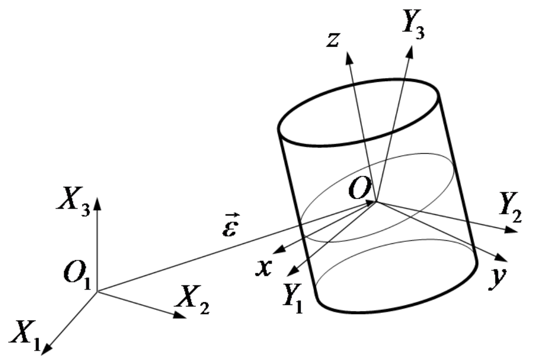

Fig. 4. Reference frames.

Let us introduce into consideration unit vectors $\boldsymbol{y}_1^0, \boldsymbol{y}_2^0, \boldsymbol{y}_3^0$ of the reference frame $OY_1Y_2Y_3$, unit vectors $\boldsymbol{x}^0, \boldsymbol{y}^0, \boldsymbol{z}^0 = \boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3$ of the reference frame $Oxyz$, so transition matrix can be represented as

$$\boldsymbol{y}_i^0 = e_{ij}\boldsymbol{i}_j,$$

where

$$e_{11} = \cos\alpha_1\alpha\cos\alpha_3,$$

$$e_{12} = -\cos\alpha_2\sin\alpha_3,$$

$$e_{13} = \cos\alpha_1,$$

$$e_{21} = \cos\alpha_1\sin\alpha_3 + \sin\alpha_1\sin\alpha_2\cos\alpha_3,$$

$$e_{22} = \cos\alpha_1\cos\alpha_3 - \sin\alpha_1\sin\alpha_2\sin\alpha_3,$$

$$e_{23} = -\sin\alpha_1\cos\alpha_2,$$

$$e_{31} = \sin\alpha_1\sin\alpha_3 - \cos\alpha_1\sin\alpha_2\cos\alpha_3,$$

$$e_{32} = \sin\alpha_1\cos\alpha_3 - \cos\alpha_1\sin\alpha_2\sin\alpha_3,$$

$$e_{33} = \cos\alpha_1\cos\alpha_2.$$

Expressions for components of angular velocity $\boldsymbol{\omega}$ in fixed reference frame will be the following (point above variable means derivative with respect to time t)

$$\omega_x = \omega_1 = \dot{\alpha_1}\cos\alpha_2\cos\alpha_3 + \dot{\alpha_2}\sin\alpha_3,$$

$$\omega_y = \omega_2 = -\dot{\alpha_1}\cos\alpha_2\sin\alpha_3 + \dot{\alpha_3}\cos\alpha_3,$$

$$\omega_z = \omega_3 = \dot{\alpha_1}\sin\alpha_2 + \dot{\alpha_3}.$$

Thus, aggregate of parameters ε_i и α_i completely characterizes motion of reservoir in conventionally immovable reference frame $O_1X_1X_2X_3$.

According to method from [2] mathematical model of the system "reservoir – liquid with a free surface" is constructed on the basis of the Hamilton–Osctrogradskiy variational principle

$$\delta I = 0, \quad \text{where} \quad I = \int_{t_1}^{t_2} L dt,$$

here the Lagrange function is given in the classical form as difference between kinetic and potential energies of the system

$$L = \frac{1}{2}\rho\int_\tau (\boldsymbol{\nabla}\varphi + \boldsymbol{\nabla}(\boldsymbol{\omega}\cdot\boldsymbol{\Omega}) + \dot{\varepsilon})^2 d\tau + \frac{1}{2}M_T(\dot{\varepsilon})^2 + \frac{1}{2}I^{ij}_{tank}\omega_i\omega_j - (M_T + M_F)g\varepsilon_z -$$
$$- \rho g(\cos\alpha_1\sin\alpha_2\cos\alpha_3 - \sin\alpha_1\sin\alpha_3)\int_{S_0} r\cos\theta(\xi + H)dS -$$
$$- \rho g(\sin\alpha_1\cos\alpha_3 + \cos\alpha_1\sin\alpha_2\sin\alpha_3)\int_{S_0} r\sin\theta(\xi + H)dS -$$
$$- \frac{1}{2}\rho g\cos\alpha_1\cos\alpha_2\int_{S_0}\xi^2 dS - (M_T h_T + M_F h_F)g(1 - \cos\alpha_1\cos\alpha_2) + \boldsymbol{F}\cdot\boldsymbol{\varepsilon} + \boldsymbol{M}\cdot\boldsymbol{\chi},$$

here ρ is the liquid density; τ is domain occupied by liquid; $d\tau = rdrd\theta dz$ is volumetric element in cylindrical coordinates; $\boldsymbol{g}$ is free falling acceleration; φ is velocity potential of liquid; $\boldsymbol{\Omega}$ is the Stokes–Zhukovskiy vector potential, which describes liquid motion in reservoir, which performs angular motion; ξ is elevation of liquid free surface; S is cross-section of cylindrical reservoir; I^{ij}_{tank} is inertia tensor of reservoir, determined relative to the point J; M_T and M_F are masses of reservoir and liquid respectively; h_T and h_F are displacements of mass centers of reservoir and unperturbed liquid relative to plane of unperturbed free surface of liquid S_0; $\boldsymbol{\varepsilon} = \{\varepsilon_x, \varepsilon_y, \varepsilon_z\}$ is vector of displacement of reservoir in translational motion; $\boldsymbol{\chi} = \{\alpha_1, \alpha_2, \alpha_3\}$ is conventional representation of turn angles of the reservoir relative to conventionally immovable reference frame; $\boldsymbol{F}$ and $\boldsymbol{M}$ are resultant vector and resultant moment of external forces, which are applied to reservoir relative to the point O.

According to [2], for efficient usage of Hamilton–Ostrogradskiy variational principle, it is necessary to construct expansions of the unknown variables ξ, φ, and $\boldsymbol{\Omega}$, which in advance satisfy kinematical boundary conditions. As it was suggested in [2], we assume the following expansions of the unknown variables

$$\xi = \sum_n a_n(t)\psi_n(x,y),$$

$$\varphi = \sum_n b_n(t)\psi_n(x,y)\frac{\cosh\kappa_n(z+H)}{\kappa_n\sinh\kappa_n H},$$

$$\boldsymbol{\Omega} = \boldsymbol{\Omega_0} + \sum_n \boldsymbol{q}_n(t)\psi_n(x,y)\frac{\cosh\kappa_n(z+H)}{\kappa_n\sinh\kappa_n H}.$$

Here $\psi_n(x,y)$ is complete orthogonal system of functions in the domain S_0, which can be determined from the Neumann boundary value problem with parameter κ_n

$$\Delta\psi_n + \kappa_n^2\psi_n = 0 \text{ on } S_0, \frac{\partial\psi_n}{\partial n} = 0 \text{ on } \Sigma,$$

where the second equation is non-flowing condition of liquid on reservoir wall Σ. Vector-function $\boldsymbol{\Omega}_0$ represents the Stokes-Zhukovskiy potential, which is solution of the Neumann boundary value problem for the Laplace equation

$$\Delta\boldsymbol{\Omega}_0 = 0,\ \frac{\partial\boldsymbol{\Omega}_0}{\partial n} = \boldsymbol{r}\times\boldsymbol{n} \ \text{ на } \ S_0+\Sigma,$$

where $\boldsymbol{n}$ is vector of external normal to reservoir wall Σ and unperturbed free surface of liquid S_0.

Since liquid is ideal, homogeneous, incompressible, its motion is vortex-free, then it follows from the Lagrange theorem that motion of liquid volume is completely defined by motion of its boundaries. This means that we can suppose variables ξ, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\chi}$ as independent, while variables φ and Ω are dependent ones. So, system of amplitude parameters a_n of decomposition of motion of liquid free surface into series by normal modes of oscillations $\psi_n(x,y)$ is considered as independent, while the parameters b_n and $\boldsymbol{q}_n$ of decompositions of variables φ and $\boldsymbol{\Omega}$ are considered as dependent on parameters a_n.

The procedure of elimination of kinematic boundary conditions on rigid walls and liquid free surface is stated in details in [2]. This procedure makes it possible to satisfy all kinematic boundary conditions for arbitrary number of considered normal modes of oscillations accurate to given power of smallness of amplitudes of excitation of normal modes a_n.

Namely, elimination of kinematic boundary conditions is done in the following way. We substitute decompositions of unknown variables into kinematic boundary conditions, multiply the obtained expressions by ψ_p and integrate the obtained relation over S_0. Here we decompose hyperbolic functions into series with respect ti ξ in a vicinity of $\xi = 0$ and later perform integration. After implementation of the mentioned procedure we obtain definite forms of dependence of coefficients of decomposition of velocity potentials on independent variables a_i

$$b_i = \dot a_i + \sum_{n,m}\dot a_n a_m\gamma^w_{nmi} + \sum_{n,m,l}\dot a_n a_m a_l\delta^w_{nmli} + \sum_{n,m,l,k}\dot a_n a_m a_l a_k h^w_{nmlki},$$

$$\boldsymbol{q}_p = \sum_j a_j\boldsymbol{\beta}^u_{jp} + \sum_{j,k} a_j a_k\gamma_j a_k\boldsymbol{\gamma}^u_{jkp} + \sum_{j,k,l} a_j a_k a_l\boldsymbol{\delta}^u_{jklp}.$$

The coefficients, entering these expressions, represent quadratures from functions ψ_i and $\boldsymbol{\Omega}_0$ taken on the domain S_0. We succeeded to get these formulae in analytical form for arbitrary number of normal modes and with accuracy, which corresponds to obtaining final equations with required accuracy, defined by powers of smallness of the independent variables a_i. This step makes it possible to transit from constrained variational problem to variational problem for free mechanical system and write the Lagrange system of equations of the second kind.

According to [2] we write the following system of nonlinear ordinary differential equations relative to independent parameters a_i (coefficients of decomposition of perturbations of liquid free surface ξ into series by normal modes ψ_i), parameters of translational $\boldsymbol{\varepsilon}$ and rotational motion of reservoir relative to conventionally immovable reference frame $\{\alpha_1, \alpha_2, \alpha_3\}$

$$\sum_i \ddot{a}_i \cdot \left\{\delta_{ir} + \sum_j a_j A^3_{rij} + \sum_{j,k} a_j a_k A^4_{rijk}\right\} + \tag{1}$$

$$+\ddot{\varepsilon} \cdot \frac{1}{\alpha^v_r}\left\{\boldsymbol{B}^1_r + \sum_i a_i \boldsymbol{B}^2_{ri} + \sum_{i,j} a_i a_j \boldsymbol{B}^3_{rij} + \sum_{i,j,k} a_i a_j a_k \boldsymbol{B}^4_{rijk}\right\} +$$

$$+\frac{1}{2\alpha^v_r}\sum_{s=1}^3 \ddot{\alpha}_s \left\{\sum_{p=1}^3 \frac{\partial\omega_p}{\partial\dot{\alpha}_s}\left[E^{1*}_{pr} + \sum_i a_i E^{2*}_{pri} + \sum_{i,j} a_i a_j E^{3*}_{prij}\right]\right\} = \sum_{i,j} \dot{a}_i \dot{a}_j C^3_{ijr} +$$

$$+\sum_{i,j,k} \dot{a}_i \dot{a}_j a_k C^4_{ijkr} + \frac{1}{2\alpha^v_r}\sum_{p=1}^3 \omega_p \left[\sum_i \dot{a}_i (E^{2*}_{pir} - E^{2*}_{pri}) + \sum_{i,j} \dot{a}_i a_j (E^{3*}_{pijr} + E^{3*}_{pirj} - E^{3*}_{prij} - E^{3*}_{prji})\right] +$$

$$+\frac{1}{2\alpha^v_r}\sum_{p,s=1}^3 \omega_p \omega_s \left[E^2_{psr} + \sum_i a_i (E^3_{psir} + E^3_{psri})\right] + \frac{1}{2\alpha^v_r}\sum_{p=1}^3 \omega^{(k)}_p \left[E^{1*}_{pr} + \sum_i a_i E^{2*}_{pri} + \sum_{i,j} a_i a_j E^{3*}_{prij}\right] +$$

$$+\frac{1}{2\alpha^v_r}\dot{\boldsymbol{\varepsilon}} \cdot \sum_{p=1}^3 \omega_p \left[\boldsymbol{F}^2_{pr} + \sum_i a_i (\boldsymbol{F}^3_{pir} + \boldsymbol{F}^3_{pri}) + \sum_{i,j} a_i a_j (\boldsymbol{F}^4_{pijr} + \boldsymbol{F}^4_{pirj} + \boldsymbol{F}^4_{prij})\right] +$$

$$+\dot{\boldsymbol{\varepsilon}} \cdot \left[\sum_i a_i \boldsymbol{D}^2_{ir} + \sum_{i,j} \dot{a}_i a_j \boldsymbol{D}^3_{ijr} + \sum_{i,j,k} \dot{a}_i a_j a_k \boldsymbol{D}^4_{ijkr}\right] - g\frac{N_r}{\alpha^v_r}\cos\alpha_1 \cos\alpha_2 a_r +$$

$$+g\frac{\alpha^c_r}{\alpha^v_r}(\cos\alpha_1 \sin\alpha_2 \cos\alpha_3 - \sin\alpha_1 \sin\alpha_3) + g\frac{\alpha^s_r}{\alpha^v_r}(\sin\alpha_1 \cos\alpha_3 + \cos\alpha_1 \sin\alpha_2 \sin\alpha_3);$$

$$\frac{\rho}{M_T + M_F}\sum_i \ddot{a}_i \left[\boldsymbol{B}^1_i + \sum_{i,j} a_j \boldsymbol{B}^2_{ij} + \sum_{i,j,k} a_j a_k \boldsymbol{B}^3_{ijk}\right] + \ddot{\boldsymbol{\varepsilon}} + \tag{2}$$

$$+\frac{\rho}{M_T + M_F}\sum_{s=1}^3 \ddot{\alpha}_s \left[\sum_{p=1}^3 \frac{\partial\omega_p}{\partial\dot{\alpha}_s}\left(\boldsymbol{F}^1_p + \sum_i a_i \boldsymbol{F}^2_{pi} + \sum_{i,j} a_i a_j \boldsymbol{F}^3_{pij}\right)\right] = \frac{\boldsymbol{F}}{M_T + M_F} - \boldsymbol{g} \cdot \boldsymbol{z}^0 -$$

$$-\frac{\rho}{M_T + M_F}\left[\sum_{i,j} \dot{a}_i \dot{a}_j \boldsymbol{B}^2_{ij} + 2\sum_{i,j,k} \dot{a}_i \dot{a}_j a_k \boldsymbol{B}^3_{ijk} + \sum_{p=1}^3 \omega_p \left(\sum_i \dot{a}_i \boldsymbol{F}^2_{pi} + \sum_{i,j} \dot{a}_i a_j (\boldsymbol{F}^3_{pij} + \boldsymbol{F}^3_{pji})\right)\right] -$$

$$-\frac{\rho}{M_T + M_F}\sum_{p=1}^3 \omega^{(k)}_p \left[\boldsymbol{F}^1_p + \sum_i a_i \boldsymbol{F}^2_{pi} + \sum_{i,j} a_i a_j \boldsymbol{F}^3_{pij}\right];$$

$$\sum_i \ddot{a}_i \left[\sum_{p=1}^3 \frac{\partial\omega_p}{\partial\dot{\alpha}_r}\left(E^{1*}_{pi} + \sum_j a_j E^{2*}_{pij} + \sum_{j,k} a_j a_k E^{3*}_{pijk}\right)\right] + \tag{3}$$

$$+2\ddot{\boldsymbol{\varepsilon}} \cdot \sum_{p=1}^3 \frac{\partial\omega_p}{\partial\dot{\alpha}_r}\left[\boldsymbol{F}^1_p + \sum_i a_i \boldsymbol{F}^2_{pi} + \sum_{i,j} a_i a_j \boldsymbol{F}^3_{pij} + \sum_{i,j,k} a_i a_j a_k \boldsymbol{F}^4_{pijk}\right] +$$

$$+\sum_{n=1}^3 \ddot{\alpha}_n \left[2\sum_{p,s=1}^3 \frac{\partial\omega_p}{\partial\dot{\alpha}_r}\frac{\partial\omega_p}{\partial\dot{\alpha}_n}\left(\frac{1}{\rho}J^{ps}_{tank} + A^2_{ps} + \sum_i a_i E^2_{psi} + \sum_{i,j} a_i a_j E^3_{psij}\right)\right] =$$

$$= 2\sum_{p,s=1}^{3}\left(\omega^*_{p,r}\omega_s + \omega_p^{(k)}\frac{\partial\omega_s}{\partial\dot{\alpha_r}}\right)\left(\frac{1}{\rho}J^{ps}_{tank} + A^2_{ps} + \sum_i a_i E^2_{psi} + \sum_{i,j} a_i a_j E^3_{psij}\right) +$$

$$+\sum_{p=1}^{3}\omega^*_{p,r}\left(\sum_i \dot{a}_i E^{1*}_{pi} + \sum_{i,j}\dot{a}_i a_j E^{2*}_{pij} + \sum_{i,j,k}\dot{a}_i a_j a_k E^{3*}_{pijk}\right)$$

$$-2\dot{\boldsymbol{\varepsilon}}\cdot\sum_{p=1}^{3}\omega^*_{p,r}\left(\boldsymbol{F}^1_p + \sum_i a_i \boldsymbol{F}^2_{pi} + \sum_{i,j} a_i a_j \boldsymbol{F}^3_{pij}\right) -$$

$$-2\sum_{p,s=1}^{3}\frac{\partial\omega_p}{\partial\dot{\alpha_r}}\omega_s\left(\sum_i \dot{a}_i E^2_{psi} + 2\sum_{i,j}\dot{a}_i a_j E^3_{psij}\right) - \sum_{p=1}^{3}\frac{\partial\omega_p}{\partial\dot{\alpha_r}}\left(\sum_{i,j}\dot{a}_i\dot{a}_j E^{2*}_{pij} + 2\sum_{i,j,k}\dot{a}_i\dot{a}_j a_k E^{3*}_{pijk}\right) -$$

$$-2\dot{\boldsymbol{\varepsilon}}\cdot\sum_{p=1}^{3}\frac{\partial\omega_p}{\partial\dot{\alpha_r}}\left(\sum_i \dot{a}_i\boldsymbol{F}^2_{pi} + 2\sum_{i,j}\dot{a}_i a_j \boldsymbol{F}^3_{pij}\right) + \frac{2g}{\rho}(M_T h_T + M_F h_F)\frac{\partial}{\partial\alpha_r}(\cos\alpha_1\cos\alpha_2) +$$

$$2g\frac{\partial}{\partial\alpha_r}\left[(\cos\alpha_1\sin\alpha_2\cos\alpha_3 - \sin\alpha_1\sin\alpha_3)(\sum_i a_i\alpha^c_i + Hl^c)\right] +$$

$$+2g\frac{\partial}{\partial\alpha_r}\left[(\cos\alpha_1\sin\alpha_2\sin\alpha_3 + \sin\alpha_1\cos\alpha_3)(\sum_i a_i\alpha^s_i + Hl^s)\right],$$

here we introduced denotations

$$\omega_p^{(k)} = -\sum_{n=1}^{3}\dot{\alpha_n}\frac{\partial\omega_p}{\partial\alpha_n}, \quad \omega^*_{p,k} = \frac{\partial\omega_p}{\partial\alpha_k} - \frac{d}{dt}\left(\frac{\partial\omega_p}{\partial\dot{\alpha_k}}\right).$$

The system of equations (1)–(3) completely describes nonlinear dynamics of combined motion of reservoir and liquid, which partially filled it, under action of external forces and moments. The system (1)–(3) consists of $N+6$ equations of the second order, where N is the number of considered normal modes of oscillations of liquid free surface. Here the equations (1) describe oscillations of liquid free surface, equations (2) describe translational motion of reservoir, and equations (3) are connected with angular motion of reservoir. In aggregate these equations describe combined motion of reservoir with free surfaced liquid for arbitrary number of normal modes of oscillations. All coefficients of the system of equations are computed as quadratures from $\varphi_n(x,y)$ and Ω_0 and some simple mathematical expressions of these coefficients. This system has specific property, namely, it is linear relative to second derivatives of unknown functions. This property predetermines potential of simple integration of these equations by numerical procedures.

3. Construction of domains of instability and conditions of transition of system on the mode of parametric resonance

The equations (1)–(3) describe the process of development of parametric oscillations in mechanical system “reservoir – liquid with a free surface”, when reservoir moves vertically by the given harmonic law $\varepsilon_z = H_z\cos(pt)$. As it is known from theory of parametric oscillations [1], there are domains in the plane of parameters (p, H_z), when solutions of the equations (1)–(3) infinitely increase, i.e., domains of dynamical instability. Construction of domains of instability will make it possible to ascertain for what values of parameters of external kinematic excitation of the reservoir (p, H_z) the system “reservoir – liquid with a free surface” will pass into the mode of parametric resonance under the presence of small initial perturbation of liquid free surface.

Let us find initially equations of boundaries of instability domain for the system of equations (1), or, in other words, for the Faraday classical system. As it is known from theoretical investigations [1,3] in this case investigation of instability can be done on the basis of linearized motion equations in a vicinity of the first (lower) resonance. We write linearized equation for the amplitude a_1 of normal mode with the lowest frequency, i.e., the first antisymmetric mode ψ_1, under the presence of external vertical excitation of the reservoir $\varepsilon_z = H_z \cos(pt)$ as

$$\ddot{a}_1 \alpha_1^v + B_{11}^{2z} \ddot{\varepsilon}_z a_1 + g N_1 a_1 = 0,$$

and rewrite it in the form of the classical Mathieu equation, namely

$$\ddot{a}_1 + \omega_1^2 (1 - \nu H_z p^2 \cos pt) a_1 = 0, \tag{4}$$

where the following denotations are introduced $\nu = \frac{B_{11}^{2z}}{g N_1}$, $\omega_1 = \frac{g N_1}{\alpha_1^v}$ is normal frequency of the first antisymmetric normal mode ψ_1. Domain of real eigenvalues of the equation (4) coincides with domain of solutions, which increase infinitely. On the other hand, domain of imaginary eigenvalues corresponds to bounded (almost periodical) solutions. Multiple roots, which have values 1 or -1 correspond to boundaries, which separate domains of real and imaginary roots. In the case of eigenvalue 1 solution of the differential equation will be periodic with period $T = \frac{2\pi}{\omega_1}$, and in the case of eigenvalue -1 it will have period $2T$.

Thus, domains of infinitely increasing solutions are separated from domains of stable periodic solutions with periods T or $2T$. Namely, two solutions of the same period bound the domain of instability, two solutions of different periods bound the domain of stability. Strict proof of this theorem is given in [3]. It follows from the mentioned theorem that determination of boundaries of instability can be reduced to determination of conditions, under which the differential equation (4) has periodic solution with period T or $2T$.

Since existence of periodic solutions and potential of their decomposition into the Fourier series is well-known, we look for periodic solution of the problem in the form

$$a_1 = B_0 + \sum_{k=1}^{\infty} \left(A_k \cos \frac{kpt}{2} + B_k \sin \frac{kpt}{2} \right), \tag{5}$$

where periodic solutions of period T are associated with even values of $k = 2, 4, \ldots$, and periodic solutions of period $2T$ are associated with odd values of $k = 1, 3, \ldots$, moreover, the number k, by which we restrict ourselves in the decomposition (5), means the number of zone of the corresponding parametric resonance (zones of instability).

For determination of boundary of the first parametric resonance ($k = 1$) we look for periodic solution in the form

$$a_1 = B_1 \cos \frac{pt}{2} + A_1 \sin \frac{pt}{2}, \tag{6}$$

Let us substitute (6) into the equations (4) and use the Galerkin method, namely, we multiply the equation (4) initially by $\cos \frac{pt}{2}$ and later by $\sin \frac{pt}{2}$, then we integrate the obtained expressions on period $2T$. As the result we obtain a system of linear homogeneous algebraic equations relative to values of amplitudes A_1 and B_1, which, as it is known from linear algebra, has nonzero solutions only then determinant, composed from coefficients of this system, is equal to zero. If we expand determinant we obtain equation for boundaries of domain of the first parametric resonance in the classical Faraday problem

$$p = \frac{2\omega_1}{\sqrt{1 - 2\omega_1^2 \nu H_z}} \text{ та } p = \frac{2\omega_1}{\sqrt{1 + 2\omega_1^2 \nu H_z}}$$

For construction boundaries of zone of the second parametric resonance we look for periodic solutions of period T as

$$a_1 = \frac{B_0}{2} + B_2 \cos pt + A_2 \sin pt + A_4 \sin 2pt,$$

and using again the Galerkin method we obtain equations of boundaries as

$$p = \frac{\sqrt{-1 + \sqrt{1 + 2\omega_1^4 \nu_2 H_z^2}}}{\omega_1 \nu H_z}$$

and

$$p = \omega_1 \sqrt{\frac{2(5 + \sqrt{9 + \omega_1^4 \nu^2 H_z^2})}{16 - \omega_1^4 \nu^2 H_z^2}}.$$

Position of zones of the first and second parametric resonance in the classical Faraday problem in the plane $(\frac{\omega}{\omega_1}, \frac{H_z}{R})$ is shown in Fig. 5. As it is seen from figure, in zone of the first parametric resonance every small amplitude of oscillations of reservoir in time leads the system into the mode of parametric resonance. On the other hand, it is practically impossible to adjust the system into the second parametric resonance, since this occurs in very narrow range of frequencies for large amplitudes of excitation of reservoir motion H_z.

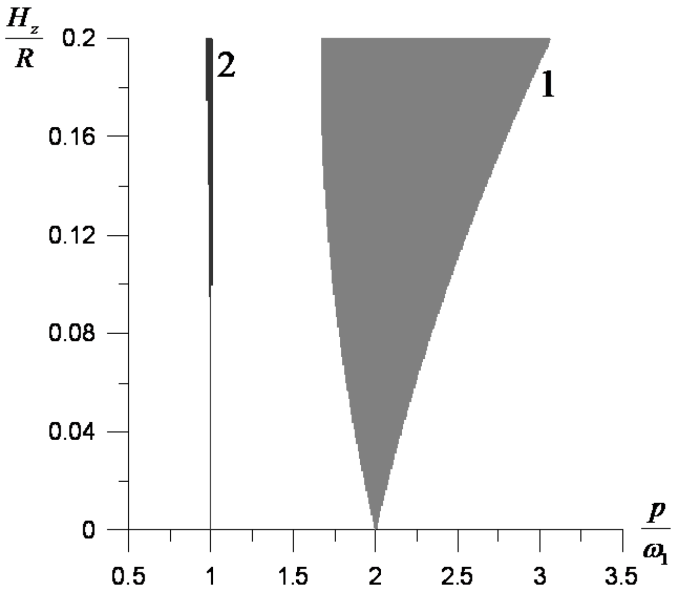

Fig. 5. Zones of the first (1) and second (2) resonance for the classical Faraday problem.

For construction of zones of parametric resonance in the generalized Faraday problem with potential of horizontal motion of the reservoir ($\varepsilon_x = 0$; $\varepsilon_z = H_z \cos pt$; $\alpha_i = 0$, $i = 1, 2, 3$) we linearize the equations (1), (2) and rewrite them only for the first antisymmetric mode ψ_1 with amplitude a_1 with potential of horizontal motion of the reservoir by coordinate ε_y

$$\ddot{a}_1 \alpha_1^v + B_1^{1y} \ddot{\varepsilon} + B_{11}^{2z} \ddot{\varepsilon}_z a_1 + g N_1 a_1 = 0,_y$$

$$\frac{\rho}{M_T + M_F} B_1^{1y} \ddot{a}_1 + \ddot{\varepsilon}_y = 0,$$

and, taking into account the denotations $\nu = \frac{B_{11}^{2z}}{gN_1}$, $\lambda_1 = \frac{B_1^{1y}}{\alpha_1^v}$, $\lambda_2 = \frac{\rho B_1^{1y}}{M_T+M_F}$, we represent them further in canonic form

$$\ddot{a}_1 + \lambda_1 \ddot{\varepsilon}_y + \omega_1^2 (1 - \nu H_z p^2 \cos pt) a_1 = 0, \tag{7}$$

$$\lambda_2 \ddot{a}_1 + \ddot{\varepsilon}_y = 0. \tag{8}$$

For construction of boundaries of zone of the first parametric resonance we look for periodic solutions of period $2T$ of the system of equations (7)–(8) as

$$a_1 = A_1 \frac{\cos pt}{2} + B_1 \frac{\sin pt}{2}, \varepsilon_y = A_2 \frac{\cos pt}{2} + B_2 \frac{\sin pt}{2}$$

and by means of the Galerkin method we obtain the corresponding equations of boundaries in the form

$$p = \frac{2\omega_1}{\sqrt{1 - 2\omega_1^2 \nu H_z - \lambda_1 \lambda_2}} \tag{9}$$

and

$$p = \frac{2\omega_1}{\sqrt{1 + 2\omega_1^2 \nu H_z - \lambda_1 \lambda_2}}. \tag{10}$$

For construction of boundaries of zone of the second parametric resonance we look for the periodic solution of period T in the form

$$a_1 = B_{10} + B_{11} \cos pt + A_{11} \sin pt + A_{12} \sin 2pt,$$

$$\varepsilon_y = B_{21} \cos pt + A_{21} \sin pt + A_{22} \sin 2pt,$$

and again by means of the Galerkin we obtain the corresponding equations of boundaries as

$$p = \frac{\sqrt{\lambda_1 \lambda_2 - 1 + \sqrt{(\lambda_1 \lambda_2 - 1)^2 + 2\omega_1^4 \nu^2 H_z^2}}}{\omega_1 \nu H_z}$$

and

$$p = \sqrt{2}\omega_1 \sqrt{\frac{5(1 - \lambda_1 \lambda_2) + \sqrt{(\lambda_1 \lambda_2 - 1)^2 + \omega_1^4 \nu^2 H_z^2}}{16(\lambda_1 \lambda_2 - 1)^2 - \omega_1^4 \nu^2 H_z^2}}.$$

Distribution of the first and second zones of parametric resonance in the generalized Faraday problem in the plane $(\frac{\omega}{\omega_1}, \frac{H_z}{R})$ for different ratio of masses ($1 - M_F = 100 M_T$, $2 - M_F = 10 M_T$, $3 - M_F = M_T$, $4 - M_F = 0.1 M_T$, $5 - M_F = 0.01 M_T$) is shown in Fig. 6 and Fig. 7. Domains of instability are bounded by curves with the same numbers.

As it is seen from figures the presence of supplementary degree of freedom results in increase of frequency of parametric resonance, moreover, the greater is mass of liquid relative to mass of reservoir (the greater is influence of liquid mobility), the greater is frequency of parametric resonance. Moreover, the greater is mass of reservoir relative to mass of liquid, the narrower is zone of parametric resonance on increase of amplitude of external excitation H_z (actually the case ($1 - M_F = 100 M_T$) coincides with results the classical Faraday problem, which corresponds to infinite mass of reservoir.

For construction of zones of parametric resonance in the generalized Faraday problem with potential angular motion of reservoir, fixed on pendulum suspension, (Fig. 3) ($\varepsilon_x = \varepsilon_y = 0$; $\varepsilon_z = H_z \cos pt$; $\alpha_i = 0$, $i = 2, 3$) we linearize the equations (1), (2) and rewrite them for the first antisymmetric mode ψ_1 with amplitude a_1 and angular motion α_1 as

$$\ddot{a} + \lambda_F \ddot{\alpha_F} + \omega_F^2 (1 - \nu H_z p^2 \cos pt) a = 0, \tag{11}$$

$$\lambda_T \ddot{a} + \ddot{\alpha} + \omega_T^2 \alpha = 0, \tag{12}$$

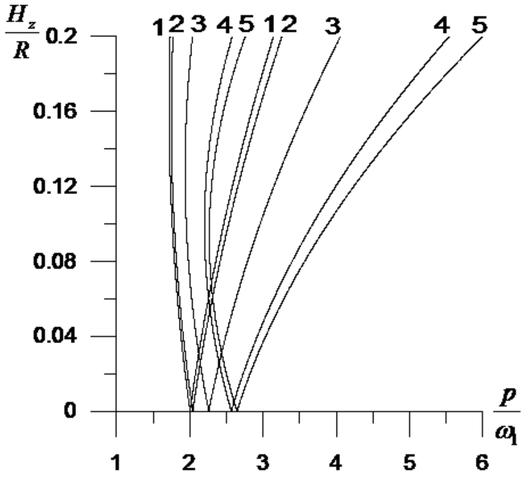

Fig. 6. Zones of the first resonance in the generalized Faraday problem with potential of horizontal motion of reservoir.

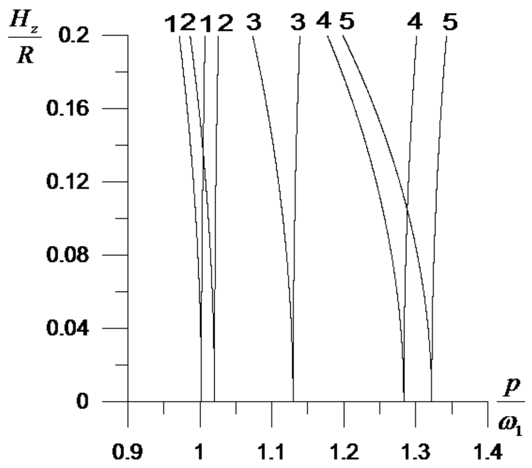

Fig. 7. Zones of the second resonance in the generalized Faraday problem with potential of horizontal motion of reservoir.

where $\nu = \frac{B_{11}^{2z}}{N_1 g}$. Zones of instability for the first resonance are bounded by periodic solutions with frequencies $\frac{p}{2}$, therefore, we represent periodic solutions for the system (11)–(12) in the form

$$a = A_1 \cos\left(\frac{pt}{2}\right) + B_1 \sin\left(\frac{pt}{2}\right),$$

$$\alpha = A_2 \cos\left(\frac{pt}{2}\right) + B_2 \sin\left(\frac{pt}{2}\right),$$

and with usage of the Galerkin method with the described above details we obtain characteristic equation for determination of zones of instability for the first resonance

$$\left(1 + 2\omega_F^2 \nu H_z - \lambda_1\lambda_2\right) p^4 + \left(-4(\omega_R^2 + \omega_F^2) - 8\omega_F^2\omega_R^2 \nu H_z\right) p^2 + 16\omega_F^2\omega_R^2 = 0,$$

$$\left(1 - 2\omega_F^2 \nu H_z - \lambda_1\lambda_2\right) p^4 + \left(-4(\omega_R^2 + \omega_F^2) + 8\omega_F^2\omega_R^2 \nu H_z\right) p^2 + 16\omega_F^2\omega_R^2 = 0.$$

Explicit expressions for solutions of these equations are not adduced here due to their awkwardness. Domains of instability for the first and second normal frequencies for different lengths of pendulum suspension ($1 - l = 100$ m, $2 - l = 10$ m, $3 - l = 1$ m) are shown in Fig. 8 and Fig. 9 in the plane "frequency of external excitation p – amplitude of external excitation H_z". Domains of instability are bounded by curves with the same numbers. As it is seen from graphs, increase of suspension length of pendulum promotes frequencies lowering and behavior of the system similar to translational motion of the reservoir. Moreover, increase of suspension length of pendulum marrows instability domains for both normal frequencies. Location of instability domain for the second normal frequency depends considerably on suspension length, but this dependence is weaker for the first normal frequency. Overlapping of this domains occurs only in the range of large amplitudes of external excitation.

4. Peculiarities of development of parametric mechanism of oscillations of liquid free surface

Previous stage of investigations was done on the basis of linear model and makes it possible to give preliminary information of development of oscillations for classical and modified Faraday problem. Actually these information gives prediction of behavior of the system "reservoir – free surfaced liquid"

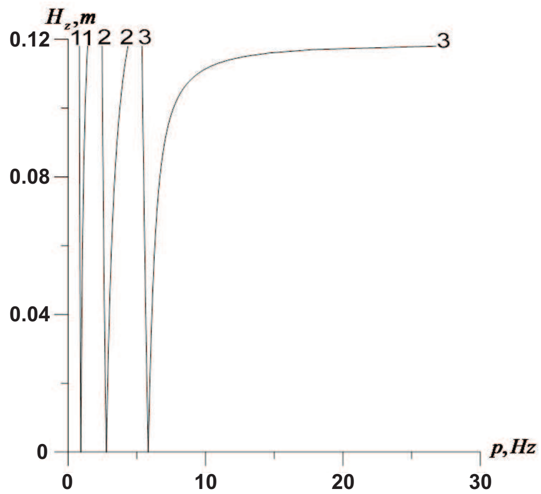

Fig. 8. Zones of the first resonance relative to the first normal frequency in the generalized Faraday problem with potential of angular motion of reservoir.

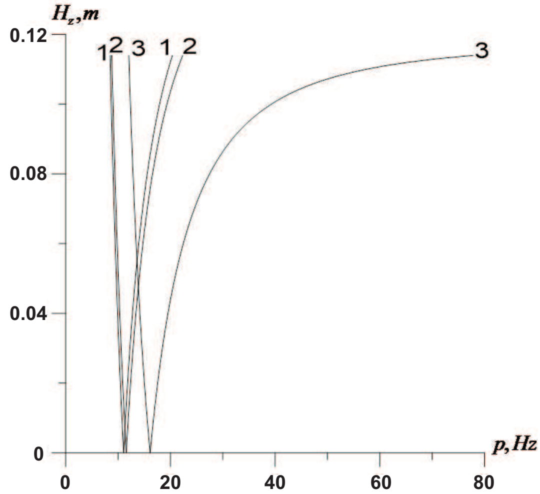

Fig. 9. Zones of the first resonance relative to the second normal frequency in the generalized Faraday problem with potential of angular motion of reservoir.

from point of view of system stability and main tendencies of manifestation of parametric mechanism in such problems. However, it is well-known that the system "reservoir – free surfaced liquid" is a system with strong manifestation of nonlinear effects and mechanisms, so, complete investigation of the system behavior should be done on the basis of nonlinear model. In this case processes will form as aggregate of corrected version of development of parametric mechanism and concurrent nonlinear processes of different nature, which originate in the system due to nonlinear properties.

Let us consider peculiarities of development of transient processes in the system "reservoir – liquid with a free surface" in the generalized Faraday problem. Actually this problem will answer the question of existence of steady modes of system behavior and forms of installation of such modes, coupled with over nonlinear mechanisms

We consider circular cylindrical reservoir with vertical longitudinal axis Oz, which performs vertical motions according to the given harmonic law $\varepsilon_z = H_z \cos pt$, and can additionally perform displacements ε_y in the horizontal plane along the axis Oy due to combined motion of the system caused by transversal oscillations of liquid free surface. Let us consider for numerical example reservoir of radius $R = 0.3$ m of M_T mass, which is partially filled by liquid of M_F mass with $H = R$ depth. For all variants we consider that initial perturbation of liquid free surface is equal to $a_1(0) = 0,01R$.

The nonlinear system of equations (1)–(2) is reduced numerically to the Cauchy normal form and later it is integrated numerically by the standard procedure of Runge–Kutta. On investigation of dynamics of the system reservoir – liquid we took into account in decompositions $n_1 = n_2 = 12$ coordinate functions accurate to squares of amplitudes and $n_3 = 6$ coordinate functions accurate ti cubic terms. Coordinate functions are distributed in ascending order of the corresponding normal frequencies except the second axisymmetric mode ψ_6, which amplitude is considered accurate to cubic terms. Step of numerical integration was selected as $\Delta t = 0,1\pi\omega_{12}$ s, where ω_{12} is the highest normal frequency in the system. On analysis of results and construction of graphs amplitudes were reduced to dimensionless form relative to characteristic size of the system, i.e., radius R of reservoir and time was normalized relative to period of oscillations of the first antisymmetric normal mode ψ_1.

Let us consider peculiarities of system transition into the mode of parametric resonance, predicted by investigation done on the basis of linear model, for the system with mass ratio $M_T = 0.01M_F$ and parameters of motion of the reservoir according to harmonic law $p = 2.6\omega_1$ and $H_z = 0.01R$. Graph of variation in time of amplitude of excitation of liquid free surface on tank wall $\frac{\xi(R)}{R}$ are shown in Fig. 10, Fig. 11 shows frequency spectrum of this excitation.

As it is seen from the graph (Fig. 10), for the given value of external parametric excitation $p = 2.6\omega_1$, which is determined according to formulae of instability boundaries (9)–(10), the system transits to the mode of parametric resonance and amplitude of excitations on reservoir walls increases more than 20 times. Here transition to steady mode of oscillations is not manifested, excitation $\frac{\xi(R)}{R}$ is characterized by amplitude modulation, which law permanently varies in time. The presence of harmonica on zero or close to zero frequencies means existence of trend (drift, nonzero mean value), which vary in wide range. We see in frequency spectrum of liquid excitations on walls dominated peak of harmonics with frequency $\omega = 1.2\omega_1 = \frac{p}{2}$, which is confirmation system transition to the mode close to parametric resonance (when dominating frequency of oscillations is twice lower than frequency of external excitation). Typical harmonics of spectrum on combined frequencies for multimodal nonlinear systems are grouped in a vicinity of dominating frequency, which is supplementary manifestation of development of parametric mechanism in the system.

For other ratio of masses of reservoir M_T and liquid M_F, shown in Fig. 6 (when relative mass of reservoir increases), in frequency spectrum of $\xi(R)$ the number of harmonics on combined frequencies close to $\omega = \frac{p}{2}$ considerable decreases, which reflect decrease of influence of liquid mobility on horizontal motion of reservoir, and in the case $M_T = 100M_F$ these combined frequencies are absent at all. So, in this case we observe pure parametric resonance as in the classical Faraday problem (oscillations of liquid have no influence on oscillations of reservoir).

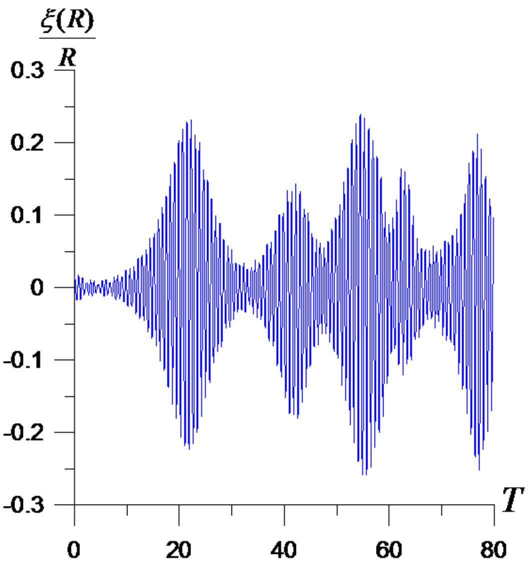

Fig. 10. Amplitudes of waves on reservoir walls on system excitation in zone of parametric resonance.

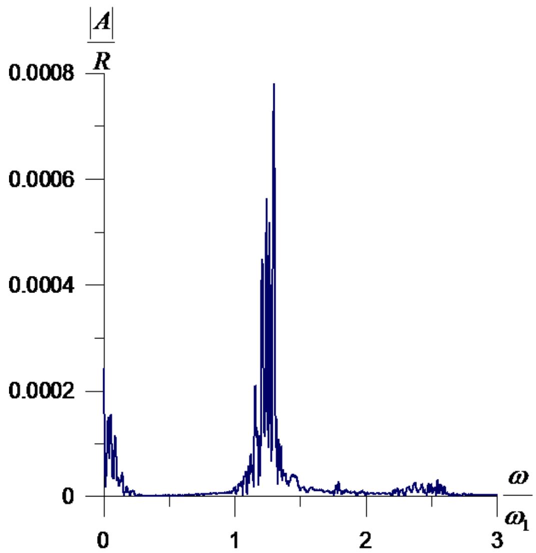

Fig. 11. Frequency spectrum of amplitudes of waves on reservoir walls on system excitation in zone of parametric resonance.

Boundaries of stability domain, specified by the equations (9)–(10) were verified also in the case, when for small amplitudes of motion of reservoir $H_z = 0.01R$ external frequency of excitation is in zone of stability, for example, below resonant frequency $p = 2\omega_1$ and above resonant frequency $p = 2.9\omega_1$ for mass ratio $M_T = 0.01M_F$. For below resonant zone of excitation contribution of parametric mechanism is insignificant (excitations of a free surface of liquid on walls are $\xi(R) \approx 0.01R$, so, they are in linear range of oscillations. For above resonant zone on the contrary considerable contribution of mechanism of parametric resonance manifests, (Fig. 12, 13), moreover, as it is seen from graphs of amplitudes and frequency spectrum on system transition to the mode of parametric resonance only transient process occurs, for which effect of high modes of oscillations is manifested considerably.

For large amplitudes and high frequencies of reservoir excitation in contrast to the classical Faraday problem dynamical processes in the system develop as aggregate of mechanisms of parametric and

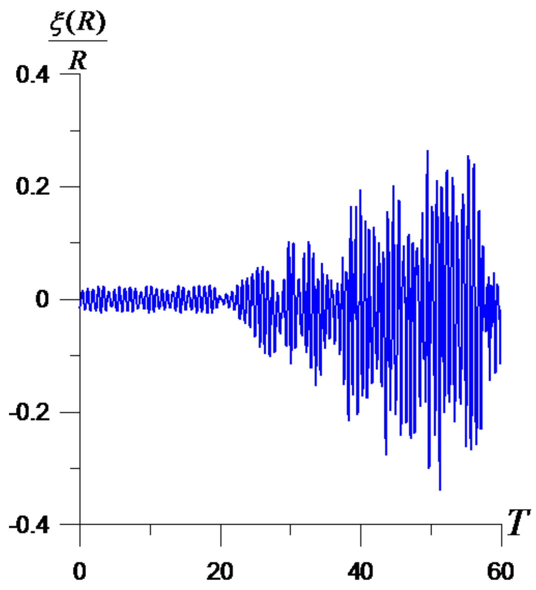

Fig. 12. Amplitude of free surface on reservoir wall on system excitation in above resonant zone.

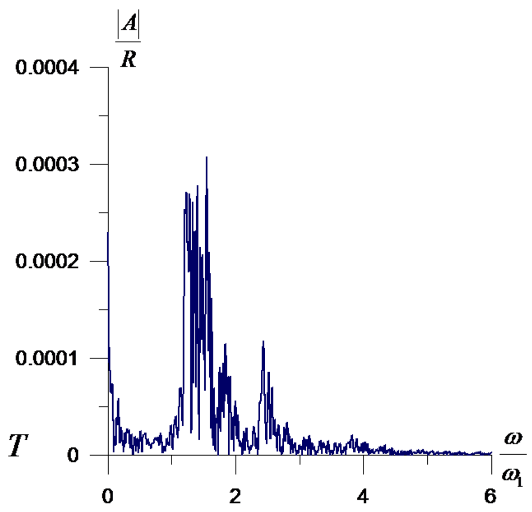

Fig. 13. Frequency spectrum of amplitude of free surface on reservoir wall on system excitation in above resonant.

forced oscillations, therefore in this case transition into nonlinear mode is possible. In Fig. 14, 15 we show results of investigation of oscillations in the system "reservoir – liquid with a free surface" for amplitude of motion of the reservoir $H_z = 0.1R$, frequency $p = 1.9\omega_1$ (below resonant zone) and mass ratio $M_T = 0.01M_F$.

In spite of the fact that frequency of external excitation is in the stability domain, mechanical system approaches to nonlinear range of perturbations. As it is seen from graph of frequency spectrum, dominating harmonic has frequency $\omega \approx p$, which means prevalence of mechanism of forced oscillations in comparison with mechanism of parametric resonance. For excitation frequency located in above resonant zone $p = 4.26\omega_1$ considerable increase of oscillations occurs during one period oscillations, here mechanism of forced oscillations dominates (amplitude of harmonic on frequency $\omega \approx p$ is twice greater than amplitude of harmonic of frequency $\omega \approx 0.5p$). Mechanism of forced oscillations promote great increase of amplitudes of oscillations of free surface of liquid even when frequency of external excitation differs from resonant one. Therefore, in contrast to the classical Faraday problem under the presence of horizontal motion of reservoir it is possible to increase oscillations of liquid considerably by vertical oscillations of reservoir.

Similar qualitative effects are observed in the generalized Faraday problem for reservoir, which is fixed on pendulum suspension and can perform angular oscillations due to transversal oscillations on free surface of liquid. Transition to the mode of parametric oscillations manifests if frequency of external excitation is from zone of instability, and initial excitation in the system can be inserted as both small excitation of a free surface of liquid and small inclination of reservoir (physical pendulum) from undisturbed position. Increase of frequency or amplitude of external excitation results in considerable increase of amplitudes of oscillations even if these parameters of external excitation are from domain of stability. Here frequency spectrum contains harmonics peculiar for both mechanism of parametric oscillations and for mechanism of forced oscillations. So, oscillations will occur on frequencies equal to frequency of external excitation or multiple to it, on normal frequencies and multiple to them ones and on combined frequencies.

We note also that insertion of supplementary degree of freedom (transversal translational or angular motion of the reservoir) in this system has different character. Translational motion does not result in increase of the number of potential resonances, while in the case of angular motion of reservoir

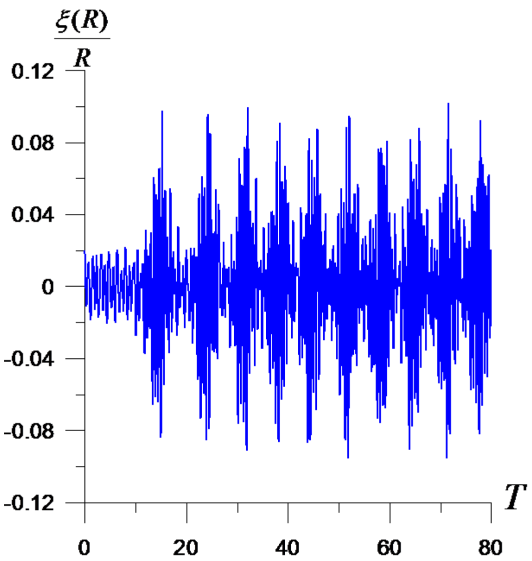

Fig. 14. Amplitude of free surface on reservoir wall for large amplitudes of excitation in below resonant zone.

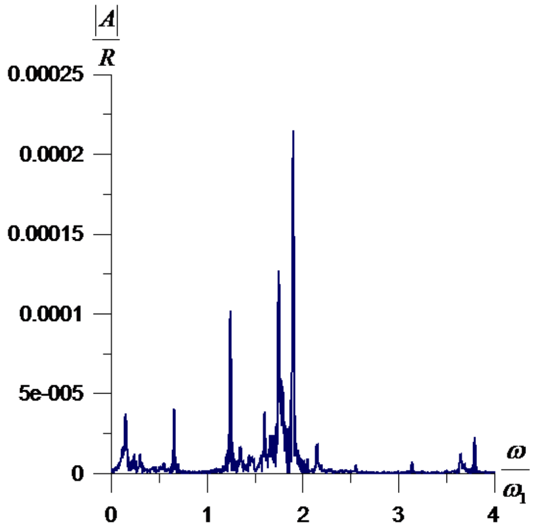

Fig. 15. Frequency spectrum for amplitude of free surface on reservoir wall for large amplitudes of excitation in below resonant zone.

supplementary frequency is generated. One potential resonance can happen for domination of angular motion of pendulum, another one corresponds to domination of liquid sloshing.

5. Conclusions

Several directions of generalization of statements of the classical Faraday problem, which are caused by the agreement of this problem with existing engineering systems, are suggested. We investigate peculiarities transient phenomena of mechanical system “reservoir – liquid with a free surface” steady mode of oscillations for the generalized Faraday problem with potential of transversal translational or angular motion of the reservoir. We investigate the classical Faraday problem with introduced supplementary degrees of freedom with considering combined character of motion of reservoir with free surfaced liquid. Study of system behavior was done on the basis of method of modal decomposition under the condition of refusal of hypothesis of possibility of neglect of oscillations on natural and combined frequencies. Introduction of supplementary degree of freedom into the system (potential of translational or rotational motion of reservoir) results in the increase in frequencies of parametric resonance, moreover, it increases with the decrease of relative mass of the reservoir in comparison with liquid mass and with the decrease of the length of the pendulum suspension. In vertical excitation of reservoir motion under the presence of supplementary degree of freedom in the system, in contrast to the classical Faraday problem, dynamical process in the system is developed as aggregate of mechanisms of parametric and forced oscillations, transition to nonlinear mode of motion with considerable increase of amplitudes of oscillations is possible for any frequency. At the same time transition to steady mode of oscillations in nonlinear multifrequency systems of “reservoir – liquid with a free surface“ type on vertical harmonic excitation of motion of reservoir does not occur, i.e. spectrum of oscillations of a free surface of liquid a;ways contain harmonics on frequency of external loading, normal frequencies of oscillations and combined frequencies, which are not multiple.

Revision of initial statement of the classical Faraday problem by considering combined character of motion of the system ”reservoir – liquid” (which better corresponds to such practical applications as rockets and launchers) and refuse form some hypothesis of linear theory of oscillations on solving nonlinear problems (which is unreasonably used in this class of problems) makes it possible to state that the classical Faraday problem is ideally academic problem, which is very narrow particular case of behavior of real systems under such types of excitations. Real behavior of reservoir with liquid under vertical kinematic excitation is much more complicated than the classical Faraday problem.

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Узагальнення задачі Фарадея в механічній системі "резервуар – рідина з вільною поверхнею"

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В роботі розглянуто два узагальнення класичної задачі Фарадея про розвиток параметричного резонансу в механічній системі "резервуар – рідина з вільною поверхнею": 1) вплив додаткового ступеня вільності – можливості горизонтального руху резервуару за рахунок поперечних коливань вільної поверхні рідини; 2) вплив додаткового ступеня вільності – можливості кутових коливань резервуару, що висить на маятниковому підвісі, за рахунок коливань вільної поверхні рідини. Дослідження виконано на основі ефективної нелінійної багатомодової моделі, яка враховує сумісний рух резервуару та вільної поверхні рідини. Показано, що на відміну від класичної задачі Фарадея, динамічні процеси в системі розвиваються як сукупність механізмів параметричних та вимушених коливань. Для розглянутих узагальнень задачі Фарадея можливий вихід коливань вільної поверхні рідини у нелінійний діапазон амплітуд при будь-якій частоті зовнішнього вертикального збурення.

Ключові слова: *нелінійна динаміка рідини, вільна поверхня, параметричний резонанс, задача Фарадея, вихід на усталений режим*

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